

1. The relativity of set-theoretic notions, and the obliquely troublemaking Skolem paradox:
There exists a countable/denumerable model which can express the statement 'R is uncountable/nondenumerable'. R can coherently be said to be uncountable in a countable model only if we interpret the cause of this as that there exists no function/relation *in the model* which maps from N to R. Thus, a set S can be uncountable in one model and countable in another. This shows that the notion of finities and infinities are *relative* to a model.
2. The extension of the paradox to the philosophy of language:
The problematic fact that the intuitive notion of a set may not be captured by the formal system extends itself to formalizations of science and beliefs in the philosophy of language. And in fact, the problem is a twofold one. Neither can our theoretical constraints (our axioms, so to speak) fix the intended interpretation, nor can our operational constraints (measurements) (the latter because we can still always have a countable model of our body of belief which meets it—assuming here that the constraints of science will only be countably many).
3. Why the problem can still not be considered to be nonexistent in set theory:
In the likely event that a complete axiomatization of set theory will not be found, there will exist sentences in set theory which have no determinate (model-invariant) truth-value. A theorem is proven which allows us to show explicitly how neither theoretical nor operational constraints suffice to determine whether an assertion (here, $V=L$, although it could just as well be AC or something else) is true or false as an axiom of set theory. (Turning it (or its negation) into an additional theoretical constraint is untenable for the realist, since he holds that the truth of $V=L$ exists as objective fact and not 'linguistic stipulation'.)
4. A clarification on the nature of operational constraints:
"Assuming here that the constraints of science will only be countably many" in #2—the existence of this assumption is acknowledged and justified. This very justification also hints us towards the following fact: That a theory dealing in counterfactuals cannot really fix any set of them to be true unless said set "is *already* fixed by something beyond operational and theoretical constraints."
5. Axiomatic truth in the middle ground between Platonism and convention:
We would be going too far in claiming that AC is a Platonic object we have access to. At the same time, we must hold that its truth is a rational, non-arbitrary fact. A resolution is found in the following perspective: The truth value of AC is dependent on the theory it is embedded in, and the theory it is embedded in is not one which can be singled out by the ultimate arbiters of rationality.
6. Scrutinizing the word 'set' to find an issue after having done so for 'axiom' and 'theory':
An analogous scrutiny exists in the physical sciences. The metaphysical realist holds that our epistemically best theory could still actually be false. But since any model of this theory satisfies all operational as well as theoretical constraints, it becomes untenable to maintain that the theory itself may be false without appealing to transcendental entities. Insofar as theoretical terms ('set') are concerned, then, we have two 'tendencies'. The realists split further: The hardcore realists (described before) and the radical pragmatists (Quine), who readily elevate the time-dependent scientific to the throne of the Platonic. But the other

wing ('the Ramsey tendency') holds that these terms are not trans-theoretic; they are *defined* by the true theories (where to be true is to have an intended model) which rely on it. (We see later than this is in line with Putnam's answer to the problem.)

7. Universal Skolemization:

Ultimately, a reformulation of Quine's indeterminacy of translation is given via Wittgenstein's rejection of a metaphysical link between our linguistic categories and our sense-datum (and a parallel of the same in cognitive psychology).

With this, we seem to have arrived at a first climax for this article. A few comments of my own are now in order. It seems to me that a clear error is embedded in Putnam's consideration of the constraints in realism. While he may agree that the list of operational constraints is jointly produced by the relevant community, it is only his tacit assumption that *observing* whether an operational constraint is satisfied is *wholly* subjective which allows him to arrive at the conclusion that two people could be trotting about with two wholly different models for the given theory. Why slice up the worlds into subjects in this particular way?

The same holds for the assumption of the pure intensionality of "mentalese" in cognitive psychology. Intersubjectivity is a fundamental part of our world.

I should add that I also think his dismissal of metaphysical realism's "mysterious mental powers" is a bit too heavy-handed and dogmatic.

8. Causality in the metaphysics of reference:

Causal chains cannot provide us with extra metaphysical strata. They merely add to our theories—more objects with indeterminate extensions. This, of course, has no effect on Skolem's argument.

9. An attempt towards truth-singularity despite the multiplicity of ideal theories:

Perhaps we can arrive at the truth by picking out all that is common between these theories. However, Putnam (and this seems here to be a mostly personal belief) feels that this would be unduly restrictive. Notably, he suggests simultaneously accepting incompatibles in an ultimate ontology for him who wishes to retrieve realism.

10. Intuitionism in mathematics:

But for him who is willing to abandon this realism altogether: If knowing the (intended, singular) truth of a theorem consists of knowing when a theorem is provable, Skolem's paradox vanishes.

11. Intuitionism in language and science:

This non-realist semantics (conceived as logically prior to realist semantics) can also be introduced to empirical science and language. "If appearance and reality end up being saying endpoints on a continuum rather than being two halves of a monster Dedekind cut in all we conceive and do not conceive, it seems to me that philosophy will be much better off."

12. Resolution:

Understanding how to use a language must mean knowing its reference. "The world does not pick models or interpret languages. We interpret our languages or nothing does. We need a standpoint which links use and reference [which metaphysical realism failed to do]."