Chapter 1

Position and momentum are sufficient to describe a classical mechanical system as far as *energy* is concerned (the physicist's equivalent of "up to isomorphism"?).

Classical mechanics is deterministic. At a given moment, it is possible to decide whether or not any meaningful property is instantiated by the state & knowledge of the laws will uniquely determine the trajectory of the state. (Reichenbach's challenge is to the second prong: The laws *can* but *need not* be deterministic, i.e. we do not have the evidence needed to conclude that the deterministic, dynamical laws of classical mechanics are the only possibility.)

Classical mechanics satisfies separability: When no causal (local and mechanical) influence is there between two systems, they are separated.

Classical mechanics is linear: The effect of the sum of the forces acting on the system is the same as the sum of their effects. (How do nonlinear classical dynamics fit into this?)

Linearity and separability imply reductionism. (Without separability, we cannot *know* how many terms there are in our reduction.) Reductionism and determinism imply the Laplacian God.

Chapter 2

The basic principles of quantum mechanics: Superposition and complementarity.

Superposition principle: If a quantum system can be in states A and B, it can also be in any normalized linear superposition of A and B.

Two features of this principle: The question of if the property A or B is possessed by the superposition state is objectively and irreducibly indeterminate; the singular state itself is delocalized and can self-interfere (whereas classically, two waves are superposed).

Chapter 3

The probabilistic nature of the superposition tells us that two systems in the same state may behave differently.

The state vector is an element of the vector space known as *Hilbert space*, which is defined over the field of complex numbers. The vector elements of this space may be called kets, and those of its dual space, bras (the are maps which map the kets back to the field C).

The form of an observable in a general basis is given. Where the form comes from will be shown later.

The projection operator is introduced. This is a linear operator is given by the outer products of each basis vector. Their sum (for a given basis) yields the identity operator. (Is this related to the completeness of Hilbert space?)

When a projection operator acts on an arbitrary state, it yields the basis vector it is an outer product of weighted by the 'amount' of the state that is in the basis vector (their inner product). Thus, it 'projects' the state onto the basis vector.

The two projection operators represent the action of the two crossed polarizers.

Therefore, a general state can be described as the sum $P_x\psi$, where ψ is the state and x ranges over the vectors in a given basis.

The basis vectors are eigenstates of the observable given by the projection operator (with eigenvalues of unity for the state corresponding to the outer product and zero for all others).

Chapter 4

Planck initially assumed not that light was *discontinuous* but, rather, that it was still fundamentally a wave which under exceptions exhibited *discontinuities.*

But after overwhelming experimental exhibitions of it, the quantization principle was formulated: Some relevant physical quantities of quantum systems can show discontinuous characteristics when the latter interact with other systems or are subject to fields or external forces. In those cases, the Planck constant is associated with the minimum quantum of energy/action involved in those interactions.

So perhaps Planck remains right in the sense that discontinuity only occurs upon interaction and not as the general default.

Historically, the shift from using functions to operators in order to describe physical quantities occurred in order to accommodate the discontinuities observed in their behavior.

While all physical observables have associated Hermitian operators, all Hermitian operators do not correspond to physical observables!

A state can be expanded in any basis, and can be said to be in a superposition only with respect to a given observable (eigenbasis). The change of basis matrix is introduced. This is a unitary matrix.

"Unitary operators effect transformations of quantum-mechanical systems in space and time." But also, "unitary transformations leave the scalar product unchanged, or invariant." Therefore, transformations in space and time leave the scalar product invariant. Is this some way of expressing the Einsteinian notion that the laws of physics remain unchanged in all reference frames? Unitary operations can also be thought of as rotations in Hilbert space (the way Lorentz transformations are rotations in Minkowski space?).

"A unitary operator represents in quantum mechanics the idea of a *reversible* transformation." Spatial changes are generally treatable as reversible. But we know that they also effect changes in time; temporal changes. How pervasive is this reversible arrow of time? Alternatively, to what extent do unitary operations capture the notion of time?

The beam-splitter is represented by a unitary operation (the Hadamard operator).

We see mathematically how introducing an obliquely oriented polarizer between two crossed polarizer reintroduces the term orthogonal to the first in the superposition and gives nonzero final intensity. This reintroduction occurs because of the "cross terms" (h)(v*), (v)(h*) in the projection operator (of the middle one). These cross terms, unrepresentable by projectors, manifest the nonlocality of quantum components, and are called *features* of quantum systems as opposed to the usual *properties*.

Also, operators do not commute. Interchanging the order of the last two polarizers would have given zero intensity again.

Chapter 5

An interesting footnote: The Kapitza-Dirac effect, wherein wave-like electrons are diffracted by corpuscular photons (coherent standing waves).

The abolishment of the matter/radiation duality of *objects* to the unified 'state' introduced another duality of *properties*: Corpuscular/Undulatory. Of course, the shift in modality means we cannot take the terms in the classical sense.

An empirical manifestation of the quantum non-separability termed as *features*, led up to by the violation of classical probabilities: Interaction-free measurement.

For seeing the violation, note merely that the probability that the photon takes the lower path (we block the upper path for this) and clicks detector D1 added to the one with which it takes the upper and clicks D1 (they are ¼ each) can easily be made by the phase shifter to be lower than the probability that it takes both and clicks D1 (superposition), whereas classically, a joint probability is always less than the sum of the individual. It is remarked that this occurs due to the extra degree of freedom—phase information—that complex numbers have.

Note that the act of blocking could arguably be called a bias, by which the classical law fails as well. This is brushed off by noting that obstacles may very well increase probabilities of detection, as we have seen previously. In any case, the force of the above argument comes from that extra degree of freedom.

If we make the joint probability zero, we have the groundwork ready for an interaction-free measurement: We can know if there is an obstacle in either path without having any photon interact with it, since the detector D1, reached by the photon using the obstacle-free path, would click only in the case that there is one!

The Mach-Zehnder interferometer analysis is redone, but this time with arbitrary transmission and reflection coefficients of the beamsplitter.

We impose the normalization condition, that the sum of their squares is unity. Next, we quantify *path predictability* by the modulus of the difference of their squares—when they are equal, we have a pure-wave like behavior (superposition) and know as little as possible; when one goes to unity, we reach the classical limit and have perfect predictability for the path.

Note that this is the same (up to a scalar factor) as the product of the probability amplitudes for final detection. Here, the product is maximum when the coefficients are most *unequal,* as opposed to classically, wherein this happens when they are most equal.

On the other hand, we introduce interference visibility as *twice* the product of the coefficients. This yields the Greenberger-Yasin equality, stating that the sum of the squares of the path predictability and interference fringe visibility is unity.

Note that the latter is also given by the difference in final detection probability (setting the phase shifter at zero). "This is understandable since the purely classical part cancels out in the difference of the two quantum probabilities."

With this, the complementarity principle is unfurled: *Local events and non-local features are complementary.*

Finally, a central question: *Where does the photon go when it is in the superposition?*

“So what is the minimum kind of reality we are allowed to attribute to this situation? The only minimum reality is the interdependence between the two components themselves.” For a position superposition, this amounts to saying that reality can only be attributed to the *relationship* between two spacetime geometries (and not to any one spacetime geometry?). Recall from Penrose that we can compare spacetime geometries by comparing geodesics in them. That article ends in the following manner: “Conventional quantum theory provides no clear answer to the problem of the stability of a quantum superposition of two different gravitating states.”

The superposition principle tells us of the existence of two modalities; the complementarity principle tells us of their aversion to each other; the uncertainty principle quantifies the lot.

Chapter 6

For the interferometer, we write the position observable as the difference between the projection operators of the upper and lower paths. Thus, the path would be eigenstates of the observable with distinct eigenvalues (equal to the coefficient of the projection operator in the observable equation—1 and -1).

This is observable is then generalized to a generic set of n possible (orthonormalized) positions (eigenstates).

The action of the observable on an arbitrary state gives us the expansion of the state in that eigenbasis, and yields the completeness relation in the eigenbasis.

In fact, for any arbitrary observable with orthonormalized eigenbasis, the completeness relation holds. The *spectral decomposition* of an observable is thus introduced. (The necessity of real eigenvalues is also mentioned. What will happen if the projection operator sum we define the observable as has a complex coefficient—entailing a complex eigenvalue? It would not be Hermitian, so the orthonormal set may fail to be a basis.)

The continuous function of x given by the inner product of the state with the position eigenstate x is called the wave function in position space.

Unlike the discrete case, the square modulus of the inner product of the state with the eigenstate x does not give the probability; indeed, the inner product would be zero. We integrate from x to x+dx and find the probability of it being found in the interval.

While in the discrete case, a state was expanded as the sum of projection operators (given by the eigenbasis) acting upon it, it is, in the continuous case, the integral of the same with dx—infinitesimal change in continuous eigenvalue.

The inner product of two position eigenstates is the Dirac delta function—infinite upon equality, zero everywhere else. Using this and the continuous spectral decomposition form, we get the eigenvalue equation.

The inner product of the variable position eigenstate with a fixed position eigenstate is called the position eigenfunction; a position eigenstate expanded in the position representation. In position space, the wavefunction of this state is the delta function at the fixed position.

Momentum is introduced as the generator of space translations. The operator for space translation by a finite distance a is derived to be exp(aD), where D is the derivative operator with respect to x.

The momentum operator is given in the position representation (its action on a position eigenstate).

The inner product of the variable position eigenstate with a fixed momentum eigenstate is called the momentum eigenfunction. Using the eigenvalue equation and the position representation of the momentum operator, we get a differential equation in terms of the momentum eigenfunction, solving which gives us its explicit functional form.

Note that eigenfunctions are always in position space.

The relationship between the momentum operator and the space translation operator is quantified. The two share an eigenbasis in the momentum eigenstates. The action of the latter on position eigenstate x is shown to be translation to the position eigenstate x+a, reconfirming our notions.

The state vector is expanded using projection operators in the momentum eigenbasis. The wave function in momentum space is introduced: The inner product of the varying momentum eigenstates with the general state vector.

The two wavefunctions are related by a change-of-basis transformation, arrived at by plugging in the functional form of the momentum eigenfunction: Fourier transforms.

The position-momentum uncertainty principle is unveiled. The canonical commutation relations are computed. It is mentioned that all observable pairs which share a common eigenbasis commute ("CSCO") (this follows from commutativity of eigenvalues, which are real).

The general uncertainty relationship between two non-commuting observables is derived. It has always interested me that the key tool used here is, somehow, the Cauchy-Schwarz inequality.

While it initially appeared that this uncertainty, caused by the shift in experimental context from classical to quantum, depended on the kind of measurements performed, we now hold that they are a deeper *feature* (indeed, the uncertainties hold even for interaction-free measurements).

Chapter 7

The correspondence principle is stated: For the limiting cases of large energies and of orbits of large dimensions, quantum mechanics passes over into classical mechanics. This entails that the classical position and momentum operators need merely be replaced by the quantum ones to obtain the correct form of the Hamiltonian.

The Schrodinger equation is presented. I want to say here that much like its classical analogue, I suspect that this can be read as ultimately merely a geometrical relationship upon Hilbert space. It is derived from the Dirac-Neumann axioms (of which one is the unitarity of time, although this chronology is inverted in the book with the unitarity of time shown here after assuming the equation) and, finally, the correspondence principle.

Using the fact that energy eigenstates are stationary (time evolution only changes its phase—separation of variables implicitly occurs *here*—separable states<-->energy eigenstates), the general problem of time evolution is considered by expansion in the energy eigenbasis. The equation for energy eigenstates is called the time-independent Schrodinger equation.

The duality of the active transformation/Schrodinger picture and the passive transformation/Heisenberg picture is introduced. Essentially, we observe how certain fixed observables of a state evolves in the former, whereas we observe how the observables of a fixed

state evolve in the latter. It is noted that commutation relations are invariant under unitary transformation.

The final result is the following: The expectation value of original observable with the state vector transformed by the operation U is equal to the expectation value of the original state vector with the observable transformed by the change-of-basis process given by U. (When U acts on the vectors of the new eigenbasis, it merely returns the old basis vectors. Thus, "the old state vector in the new basis is equivalent to the new state vector in the old basis.")

Writing the Schrodinger equation in the Heisenberg picture ('the Heisenberg equation'): The time derivative of an observable is computed. In general form, this equation is the Ehrenfest theorem.

"In classical mechanics, there is no equivalent of the Heisenberg picture;" the necessity never arose. Lastly, a third picture lying between the two ('the Dirac picture', based on the split between interacting and non-interacting parts of the Hamiltonian—with other systems, that is) is mentioned.

The energy eigenvalue (expectation value of the Hamiltonian) is always greater than the minimum potential value (this can be seen by replacing the Hamiltonian with the momentum and potential operators).

For a free particle, V(x)=constant everywhere. We set it to 0 and solve the time-independent Schrodinger equation, giving the energy eigenfunctions, and compute the normalization constant.

(The *actual* free particle in infinite space is not an energy eigenstate, and not normalizable. Here, it has been taken between finite bounds; a sort of finite potential well.)

Some salient points: Only energy eigenstates satisfy the TISE, while all states satisfy the TDSE.

Each position of the free particle corresponds to a variety of energies/energy eigenvalues (ground state, excited states…).

One can also see by direct comparison with the momentum eigenfunction that this energy eigenfunction is also a momentum eigenfunction, with eigenvalue hbar times k. In other words, this Hamiltonian and the momentum operator commute.

Furthermore, each energy eigenvalue corresponds to *two* momentum eigenvalues/eigenstates (and also *two* energy eigenstates)—degeneracy. "The condition for a degenerate energy spectrum is that there exists at least one nontrivial observable that commutes with the Hamiltonian of the system." (Rationalize it as: Energy has been measured to be at a certain eigenvalue; the other observable is measured and could be yield of various eigenvalues; thus, the system is in one of various possible eigenstates for that energy eigenvalue.)

The harmonic oscillator is discussed and solved using creation & annihilation operators.

The *density matrix* is introduced as a generalization of the concept of projectors (outer products).

The density matrix of a pure state written not in its eigenbasis has (here, for a two-dimensional space) the classical coefficients along the diagonal and the quantum features along the off-diagonal. That pure state is an eigenvector of the density matrix with eigenvalue unity (assuming it is normalized).

The generalization part comes when one considers the density matrix of what is called a mixed state. In this case, we do not *know* the quantum state we are dealing with; it is either the horizontal state or vertical state with 50-50—not as superposition but in the classical sense.

In this case, the density matrix has the classical coefficients along the diagonal and zero along the off-diagonal. In general, the density matrix is the weighted sum of the projectors of the pure states our quantum state may be in.

The expectation value of any observable for a mixed state, defined by the weighted sum of its expectation values for the pure states, equals the trace of the matrix given by the action of the density matrix upon the observable. Some further properties of a density matrix are that it equals its transpose, its trace is always unity, its expectation value for any state is nonnegative (the matrix is positive semidefinite), and that the density matrix of a pure state equals its own square.

We can also write, using the Schrodinger equation, the equation governing the time-evolution of a density matrix. This is referred to as the *von Neumann equation.*

The final and most important point requiring explication regarding this fresh entity is its relationship with the quantum state. While the matrix itself is an observable, the curiosity lies in the fact that it *represents* a quantum state. In other words, the result of measuring this observable tells us something like “Yes, the system is in this quantum state” or “No, it is not in this state”.

But note even the semantic difference here—the fact that it is not the same *sentence* as “Yes, the system is (in the quantum state) horizontally polarized, spin-up, position x=2, momentum 3 units”. The point is that measuring this observable does *not* provide us with some overarching information about the state of the system. The confusion arises because one erroneously presumes in the discussion that one knows all the information about the quantum state you are asking the observable if the system is in, and that a simple ‘yes’ can thus fill up all the lacunae.

If the density matrix observable corresponds to a mixed state, measuring it means measuring something described by the one or more of the projectors constituting it (think of an example, say, a mixed momentum state). Since the outcome of this measurement only provides partial information (as with all quantum measurements), it is impossible to know the state of a system (or, therefore, to distinguish between non-orthogonal states) in a single measurement.

In clearer terms, measuring an observable for a system gives us only one of its eigenvalues, and not anything more about its other properties for the same (for example, measuring the position observable does not supply us with information regarding its time derivative—in other words, the non-commuting momentum observable).

A Hilbert space corresponds to a system. The “direct product” (and I think this is the tensor product) of two 2D Hilbert spaces corresponding to (here) two distinct two-state systems yields a 4D Hilbert space of the composite system. Let us characterize it formally.

The basis of this space is given by all the possible direct products of the bases for the two 2D spaces—in this case, four. The space itself is the span of this basis.

The inner product of two composite states is then defined (in terms of the constituent states). It seems that it is this inner product definition that finally allows us to call the direct product basis an orthonormal basis, give them a component form, and conclude that the resultant Hilbert space is of four dimensions.

First, the composite state formed by the direct product of two arbitrary superposition states (one for each system) is discussed. This is separable and quite straightforward.

Next, quite a different-looking composite system is written (one which quite clearly cannot be written as the direct product of two pure states, unlike the above example). Of course, this is the composite state of two *entangled* systems.

The authors refrain from really discussing the nonlocal dimension of the situation for the moment. Instead, we consider the outer product (the density matrix) of the given composite state (which, note, is still a pure state).

In this direct product we can once again see two interference terms/quantum features. (Contrary to appearances, they do not stand for states of different systems but, rather, different states of the same composite system.) "Therefore, entanglement can be understood to a certain extent as an extension of the concept of superposition to a composite system."

There is a crucial difference, however, in that superposition is relative to a basis, while entanglement is an intrinsic property of the state (an entangled state remains an entangled state regardless of which basis we write it in). This is because "entanglement combines the typical quantum superposition with correlations that have a classical root." (Perhaps one can think of the classical aspect as inducing basis-invariance, since we know that classical correlations are basis-invariant by nature.)

What happens when we try to reduce the entangled composite into its subsystems? The *reduced density matrix* (for, say, system 1) contains only information about system 1 and is obtained by "tracing out" the system 2 ($Tr_2 * \rho_E$)—or, alternatively put, by summing over the expectation values of the density matrix over the orthonormal basis of system 2.

These formal characterizations are not really expanded upon. But in conclusion, a bipartite pure state is entangled if and only if its reduced subsystem states are mixed. Thus, this becomes an important example of why density matrices are important; their use here is unavoidable.

Chapter 8

Akin to the generator of space translations, the generator of rotations about the z-axis is introduced, followed by the rotation operator about the z-axis; finally, their relationship with the angular momentum operator is computed analogously.

Mention is made of the Noether theorem in order to justify the fact that for every kinematic parameter (such as angular momentum), there is a dynamical quantity which acts as its generator (such as D and R for p and L respectively).

The correspondence principle followed by some elementary mathematics gives us the canonical commutation relations between the A.M. operators; none of the A.M. operators associated with one of the three standard axes commute with one another, but all of them commute with L^2.

Now let us focus on Lz.

Since it commutes with L^2, they have some common eigenstates. We postulate the eigenvalue equation of Lz for these, each state being characterized by *two* parameters—l, m—corresponding to each of the two operators.

Using the fact that the expectation value of the square of a Hermitian operator is nonnegative, it is shown that m \leq |l|. l is christened the *azimuthal quantum number.*

Raising and lowering operators are introduced.

The action of the former on an eigenstate is to turn it into another state with eigenvalue m+1. It follows that the state given by the action of a raising operator on l, m must be proportional to the state characterized by l, m+1.

Correspondingly for the lowering operator.

Their actions on the eigenstate with the maximum and minimum magnetic quantum number *m* is set to zero.

By now ordering all the eigenstates for a given l (called a "multiplet"), we see that l must be of the form n+0.5. The half-integral values of l are associated with *spin;* for orbital/rotational motion in real space, l is a nonnegative integer. m can take up the associated 2l+1 values.

The orthonormality & completeness conditions are written out for all these eigenstates common to L^2 and Lz.

With some quick mathematics, the eigenvalue equation for L^2 is procured. It is slightly *greater* the square of the maximum Lz eigenvalue, although in fact, one can see that if the A.M. operators commuted, then it would be equal to this square (or indeed, equivalently, if the expectation value of the squared operator of both the others was exactly 0; for this means their eigenstates are exactly known to be ones with eigenvalue 0). In fact, what this tells us is the following: "The angular momentum and its z component can never be aligned". (z could of course equally well be x or y.)

Having derived the eigenvalue equations, it is time to look into the eigenfunctional form.

Rewriting everything in spherical coordinates, we see that the A.M. operators leave the radial component unaffected. We thus extract the same out of the eigenfunction. These are called the *spherical harmonics.* Analyzing the same gives us back the integer-value condition for orbital l. The eigenstates of L^2 for l=0,1,2,3 are what we call the *sharp, principle, diffuse* and *fundamental* (s,p,d,f) waves.

The hydrogen atom's time has finally come.

We have the Coulombic potential. The Hamiltonian is written in spherical in terms of the L^2 operator. The energy eigenfunction is a continuous multiple of the spherical harmonics, the multiplicative factor being a radial function. We plug it into the equation and perform a change of variables for the sake of simplification (using the facts that the RHS will be Ef(r) and the action of L^2 will be f(r) times the eigenvalue of the spherical harmonic).

The equation is now reminiscent of the Schrodinger equation for a particle with a potential slightly raised from that of the Coulombic alone (due to the term given out by the action of L^2 on the eigenfunction). This excess 'effective' potential is called the *centrifugal potential barrier,* and its form being evidently repulsive, is what prevents the electron from falling into the center. *This* is where the Schrodinger equation gives the electron the 'extra' force/energy keeping it in place. The extra effective potential, it is interesting to note, depends on l, 1/m and 1/r^2. For l=0, the barrier vanishes, since the electron is not rotating at all. Thus, what is later called the Larmor effect is avoided. The discreteness of transitions is also expressed in the discreteness of l.

The differential equation is subsequently solved using the normalization boundary condition. We see that the radial function is determined by n, l; the overall energy eigenfunction, by n, l, m; and obtain the known formula between the energy eigenvalue E and n by reversing the change-of-variables (n

was, in fact, just an appropriately defined change-of-variables back when the equation was being solved). Since the eigenvalue remains the same for a given n as we change the eigenfunction by varying l, m over their respective allowed values, we see that there is degeneracy. To be precise, we see from the n-1 possible l and 2l possible m that there must be n^2 degenerate eigenstates for a given eigenvalue.

The effect of *spin* angular momentum was first observed in the following manner: Silver atoms were sent through a strongly inhomogeneous magnetic field, and their deflections was observed.

But the orbital angular momentum of silver atoms is 0, and thus, so is the magnetic momentum due to it. Thus, the second degree of freedom inducing magnetic moment, *spin,* entered the picture. (Second degree of freedom because it is not the case that either l=1 or l=0.5; we have *both* l=1 and s=0.5 or even l=1, s=1).

The commutation relations and eigenvalue equations are entirely analogous for the spin operators.

A few heavyweights are namedropped: Pauli matrices & spinors. Not much is said about them, however.

The relationships between the former and the spin operators is stated. Unlike the orbital angular momentum eigenfunctions, there are straightforward relations between the eigenstates of each dimension's spin operator (all of this for s=0.5) (presumably from the Pauli matrices, although it is not really explained).

The spinors are just the (columnar) representation of the wavefunction in position space after the additional degree of freedom due to spin has been included. In the same vein, the hydrogen atom energy eigenfunction will also have an added factor (the spinor) for the additional degree of freedom due to the presence of the electron's spin (separated out, since the Coulomb potential is independent of spin). Degeneracy goes up to 2n^2.

(Why only two? Why not an electron with z-spin up, z-spin down, x-spin up, and all the other infinitely many superposed states? These are all still distinct states…degeneracy blows up to infinity! Well, because only two are linearly independent. From definitions: An eigenvalue λ which corresponds to two or more different linearly independent eigenvectors is said to be degenerate.)

Just the way we related the orbital angular momentum operator with the generator of rotations in real space, we connect the spin angular momentum operator with the generator of rotations in *spin space* (which, it seems, also has the directions x, y, z…not so different from real space after all? I think the analogue is misleading.). Upon computing its form for the s=0.5 case and replacing the operators with the associated Pauli matrices, we see that the latter are, in fact, the respective rotation operators in spin space by an angle pi about their respective axis.

Using the general rotation-generator operator, we may write the spin-state along any arbitrary direction by applying this operator (say, by first rotating about z and the next, about y) on a z-axis spin state. The operator along the axis corresponding to this spin-eigenstate is given. We can see that it gives back the familiar Sx, Sy, Sz forms upon plugging in the right angles into the rotation-generator operator.

Finally, a Mach-Zehnder interferometer style experiment on how we may experimentally realize the above procedure.

We are left with 4 numbers quantifying the particle's angular momentum. We can, in fact, 'add' any combination (ordered pair) of these.

Let us, for the moment, restrict ourselves to finding the sum of two spin 0.5 particles. We calculate the total possible sums for m(s) and backtrack to find the possible totals for s. (This is because we are restricting ourselves to the z-axis, and the angular momentum operator along a given axis will only pull out m, giving us eigenvalues which are the sums of the two m. It follows that s is not just s1+s2, since that would unduly restrict the possibilities for m.)

The corresponding common eigenstates are written for each sum-pair (as a tensor product). There will be three for s=1 and one for s=0. It is noted that one of the triplet states, as well as the singlet set, are entangled ones.

Two particles with the same "intrinsic physical properties" are indistinguishable (although the properties are not rigorously enumerated—there lies an important question of identity here which has not been explored.) The product state of two particles which can occupy one of two possible states each should remain unchanged if said particles are exchanged: A symmetry.

Well, not exactly—one of them can still possess an extra phase factor. (Presumably because indistinguishability was characterized in terms of those certain physical properties which remain unchanged upon appending phase factors.) We thus obtain two solutions for the product wavefunction—symmetric and antisymmetric. The class of particles producing the latter state have half-integer spin and are called fermions; the others have integer spin, and are called bosons.

Furthermore, *all* particles are either fermions or bosons. The nature of the composite particle can be deduced using the rules of angular momentum addition discussed previously.

If the two states are the same, the antisymmetric state vector becomes 0. This yields Pauli's exclusion principle: *No two identical fermions may occupy the same state.*

Often, this forces fermions into higher energy levels in order for them to distinguish themselves, leading to *degeneracy pressure*.

PART III

Chapter 9

There is no such thing as knowing a problem and not knowing its solution—must not knowledge of the solution be part of full knowledge of the problem?

And so, let us see what exactly the measurement problem is.

1. Time-evolution is unitary
2. Measurement is non-unitary

That is about it.

We have gone over the former; let us look into the latter.

It seems to be a bit difficult to make sense of the fact that measurement is non-unitary; certainly, one can always construct a unitary transformation between a given superposition state and any one of its eigenstates.

The point is that in all transformations that may be referred to as 'measurement', we do not *know* exactly what the form of the superposition is (for otherwise, it is senseless to say anything is being

measured). And quite evidently, there is no single unitary transformation between a general unknown superposition state and an eigenstate.

But how can *our* lack of knowledge have any effect?

One way of reasoning this out I saw is that measurement *selects* a unitary (after we measure the state and figure out precisely what the initial superposition was), rather than *implementing* a unitary.

Another way to look at it is by entanglement. The first step of measurement is (the unitary process of) entangling the system with the apparatus. But measurement itself occurs only for one component of the whole entangled system; this leads to the rank-increase of the density matrix/projective non-unitary action. We shall see this notion of non-unitarity due to globality in decoherence.

von Neumann postulated a certain form for the post-measurement state with density matrices and projects, calling nonunitary projection-like transformation the "reduction of the wavepacket", something which is random, irreversible. This reformulates to a classical probabilistic sum of the eigenstates when we take the partial trace/reduced subsystem of the quantum state after considering its entanglement with the apparatus—which makes von Neumann's final state a mixed state.

But how do we get the actual determinate result out of this? Either due to the power of the mind, which is the subjectivist interpretation (which considers the von Neumann final state as a sort of mysterious in-limbo transit, perhaps akin to the state of a classical coin before it becomes heads or tails?)—or something else happens: Let us look into the objectivist interpretation a bit, formulated in Everett's many-worlds theory.

The objectivist interpretation reduces von Neumann's reduction to an illusion, and claims that the final state is the entangled unreduced state all along, which is called the *relative state*; and that we perceive only one component of it is because of the "perspective" we adopt to view it; and that as such, all of them are equally real.

One difficulty with this, known as basis ambiguity, is explained.

Suppose we are measuring some generic observable which the state is in a superposition of. Let the apparatus itself also be in some state A0, which then becomes entangled with the quantum state we are concerned with.

But now we can rewrite the states of the apparatus (each component corresponding to the apparatus in the overall entangled state) in another basis. Substituting this expansion back, rearranging and finally absorbing constants, we get another expansion of the same entangled state—which is to say, an expansion of another observable entangled with the apparatus.

This would seem to imply that we can measure multiple observables in one measurement—and these observables need not even commute, in general.

If we try and escape by letting go of the interpretation that a measurement is intended for measuring a *particular* observable in *all* worlds, and say instead that *all* observables are measured in *particular* worlds (that is, observable O1 in world 1, O2 in world 2, etcetera), then the idea of measurement seems to lose all significance.

It would appear that Everett misses the distinction between (mathematics and dynamics, or) *preparation* (which just pertains to getting the quantum state in the superposition in question) and *premeasurement* (which entangles the apparatus with the state, and imposes physical constraints which fix the observable being measured—constraints which are incommensurable with the finality of the relative state)—although since premeasurement is still reversible, Everett is correct with his theory to that extent.

Decoherence theory and the role of the environment: Let us consider for a moment the fact that the state-apparatus entanglement will also be entangled with the environment—indeed, with all the systems in the whole universe ("more or less entangled"—entanglement being a matter of degree will be explained later-on).

But in the act of measurement, we localize ourselves to the apparatus and the state, neglecting the environment. Representing this mathematically amounts to tracing out the environment from the overall entangled state—but this is us nothing but the classical mixture of two eigenstates!

The original entanglements have not disappeared but are "lost in the universal quantum noise background of our universe". Coherence across components has decreased; hence the name *decoherence.*

This leads us to postulating a crucial juncture: Global reversibility (at the level of the universe, all things go as per quantum mechanical dynamics) and local irreversibility. This, it is said, is a distinction between locality and nonlocality which is unknown to classical physics.

We shall utilize the concept of entropy in order to further explore this idea.

But in order to apply the correspondence principle, we must recognize the fact that classically, there are two kinds of entropy: Shannon entropy (information entropy) and Boltzmann entropy (thermodynamic entropy).

The former quantifies the amount of information we may glean from a certain process (if the probability of a given occurrence after said process is unity, then the information obtained is 0). The entropy itself "is defined by the sum of the information contents over all the possible outcomes weighted by the corresponding probabilities".

I am already familiar with the classical Boltzmann entropy.

A relationship between the two is shown (the Boltzmann entropy for an isolated generic gas system equals the maximum value of the Shannon entropy for the same, times a factor). It is also mentioned that the true counterpart of the Shannon entropy in thermodynamics is the Gibbs entropy, which reduces to the Boltzmann entropy for equiprobable microstates (we have seen Gibbs' paradox in Reichenbach's book with relation to it).

It is clear that both entropies represent 'disorder', in the sense that an increase in either entropy has an equivalence with an increase in the system's disorder.

One may further associate a Shannon entropy *to* a Boltzmann entropy, since an increase in the former indicates an increase in the amount of information it is possible to acquire.

It is finally emphasized that the respective entropies are neither the same as information (it is a formal quantity which is perfectly possible to reversibly process—Shannon entropy enters the picture in those cases when the process turns irreversible, due to disorder—we can see that

Shannon entropy is minimum/0 when the (classical) probabilities are either zero or unity with no disorder), nor the same as heat.

Now, coming to quantum mechanical entropy, known also as *von Neumann entropy*: This is defined mathematically (in terms of density matrices). Using the fact that density matrices are diagonalizable, it is reformulated in an expression reminiscent of Shannon entropy. Notably, the von Neumann entropy is zero for pure states.

Measurement can now be postulated as a local increase in entropy which leaves global entropy invariant (zero) via to the loss of certain features of the local system into the environment (causing a decrease in entropy elsewhere).

Entanglement is the unitary transfer of information from the object system to the apparatus. It is easy to see that after this, *we* can acquire information only if the states of the object system are orthogonal. "Only this requirement can reconcile the linearity of preparation and premeasurement with the non-linearity of the final selection," for this means that the apparatus functions only when this requirement is satisfied, when the components of the object system coupled with the apparatus become orthogonal—a requirement which can only be satisfied non-unitarily (this is easy to see—for example, the Gram-Schmidt orthogonalization process is clearly not unitary).

Let us formalize the above postulate.

Theorem: For any local irreversible transformation of a given quantum system it is always possible to choose a larger quantum system that embeds it and in which the transformation is reversible.

Proof: Consider a mixed state, the outcome of some irreversible transformation. Consider also a composite state, wherein an 'environment' state-vector component is entangled with the states of the mixture. Tracing the environment out of the latter gives us back the mixed state, and this proves the theorem.

A brief introduction to logic gates is given. Considering the two-input case: Conjunction is reversible only when it is true (when it is false, it has 3 pre-images). In general, even the other operations are irreversible. It is then shown how one may embed an irreversible transformation in a reversible one.

Finally, a brief exposition on the zero-entropy universe and the distinction between *subjective* and *relative*.

Chapter 10

The EPR argument: There is no real need to delve into the hardcore ontology of what they may mean by physical reality. They essential point is acceptable even when viewed pragmatically: It is evident by the argument that incompatible observables do *exist* simultaneously, even in the minimal operational sense—and this is all that is really required.

Take a momentum-entangled composite particle system and measure the momentum of 1. We can then conclude that the momentum of 2 exists (and can know its value, in fact).

Rewriting the composite system in position eigenfunctions, we see that the positions are also entangled. Measuring the same of 1, we conclude that the position of 2 is also an element of physical reality.

But since we have ensured that the two systems are *separated*, it follows from the fact that we have only interacted with system 1 that the properties of system 2 remain unchanged. And so, both position and momentum are elements of physical reality—but since quantum mechanics can describe only any given one, it was concluded that QM is incomplete.

Bohr criticized this argument by attacking the first assumption—the criterion of physical reality—saying that since the 'context' of measurement for momentum-1 and position-1 differ, so does what can be called a 'physical quantity' (and that, I suppose, momentum-2 ceases to be called a physical observable because of the *kind* of measurement the second one is).

Bohr's point fails due to the epistemic context in which he made it (and thus, does not rebut the EPR argument). What is true is the ontological statement that an entity can only be called a physical quantity with reality if a possible measurement for it *exists*.

But the next step, which is that not only the observable but also an eigenvalue of it is real, is what we need an actual measurement for. This was where Schrodinger formalized EPR's raw notion of entanglement and rebutted them correctly (essentially taking down the other prong, separability, of the EPR argument).

Bell's Theorem: A deterministic local hidden variable theory, which acknowledges the separability principle, must satisfy the Bell inequalities. The predictions of quantum mechanics violate these inequalities.

For ease of analysis, we consider a spin-entangled state (the singlet state).

The nub of the argument is that, for such a theory, the quantum-mechanically determined expectation value for the given state will not, in general, equal the classical statistical expectation value via the hidden variables.

In fact, Bell shows explicitly in his superb original paper that they can indeed be made to agree for non-entangled pure states. It is in *this* case, with entanglement and non-separability, that the Bohmian theory would fail. "The statistical predictions of quantum mechanics are incompatible with separable predetermination."

In the next section, it is shown how one may, given 4 polarization-entangled photons (1-2, 3-4 respectively), 'swap' their entanglements to (1-4, 2-3). The catch is that 1 and 4 never need to meet at all; for example, we send 1-3, 2-4 in opposite directions and need only to measure the Bell-operator observable for, say, 2-3. This shows that entanglement is not dynamical, nor associated with causal relations.

Since there are no causal relations involved in entanglement, we are able to maintain the Einsteinian postulate that no causal signal can travel faster than light. It is now time to formalize this and distinguish precisely via Eberhard's theorem between *separability* and *locality*: For it is an absence of the former in quantum entanglement.

Eberhard's theorem: Quantum mechanics does not allow non-local correlations between apparatus settings.

Mathematically, this is not too difficult. Consider a pair of entangled observables, O1 and O2. Using density matrices, it is shown that the probability distribution of the second observable's outcomes after the first has been measured is the same as its probability distribution before the first has been measured.

But—what? We over at O1 know that O2 will get a certain outcome with certainty. How, then, can we say that O2's probably distribution is the same? Does this not merely reflect a lack of information for the measurer of O2? Well, yes—and I suppose that is precisely the point. The change affected is not dynamical, preserving Einstein, but some constraints of a hitherto *new* kind, which is what we call non-separability. And I remain unable to truly wrap my head around it.

Onto the third theorem of this chapter.

A statement d is given, composed of 10 atomic statements. It is represented with the 10-point Kochen-Specker graph, and a lemma about it is proven (if d and a0 are true, so is a9).

The statement s' is constructed using 15 copies of d. Three groups, each one of five interlocking copies. Describing the group structure: Each group has one common a8 along the five interlocking copies, and the a9 of one copy acts as the a0 of the next. Then, these 3 groups are brought together by identifying the a8 and the last a9 of one group with the first a0 of the next next and the next group respectively. The three a8s (one for each group) form a triangle.

This 117-point Kochen-Specker graph is the statement s'.

Let d' be d without b08. We finally write out s'.

In fact, it is said that this statement is the "opposite" of the statement s, which reads "When the operators describing two physical quantities do not commute, the two quantities cannot have simultaneous reality."

Kochen-Specker theorem: The statement s' is classically a logical falsity, but there are cases in which it is true in quantum mechanics.

That it is classically false is proven by a straightforward reduction ad absurdum.

And that it can be true in quantum mechanics is shown with an explicit counterexample.

Now, the bottom line here is that there exist value assignments in quantum mechanics which are not "meaningful" classically: We cannot have classical value assignments for all eigenvalues of observables.

In other words, while Bell shed doubt on separability, KS shed doubt on realism. Thus, the KS argument is, in fact, a refinement, an ontic generalization, of Bohr's original rebuttal: The former sheds doubt on the ascription of reality to *properties* altogether, for it excludes the very possibility of any eigenvalue-assignment, while Bohr only claimed that eigenvalue-assignment was context-dependent.

In fact, ironically, Bohr's argument has turned out to be one way of escaping the KS argument. For we may say that we can still have such a classical value assignment, provided it is *contextual*. This retrieves realism by separating it from an assumption typically intertwined with it (namely, that measurements are 'faithful'), and essentially falls back to the option of nonlocal realism.

(For imagine if we measured a0, and it turned out to be true. We start making our classical inferences and end up at the conclusion that a0 is false—but in fact, this will not be a proof of the classical falsity of s' by contradiction; rather, we say that a0 is, indeed, now false, because of the different 'context' in its second measurement.)

Chapter 11

The unique importance of information: To have both non-separability and locality, one must have *unphysical correlations.* This marriage of something physical and something formal is what is found in the notion of *information*. "What is interesting about information is that it is a quantity 'sitting' somehow between pure mathematics and physical reality. It must necessarily be instantiated in some physical media but, as we have seen, both the information context and its syntax are not dependent on the specific physical characteristics of the medium."

The Bloch sphere (sometimes known as the Poincare sphere) is introduced. This is a way of representing any quantum state in a 2D Hilbert space (a qubit/quantum bit). The two degrees of freedom are quantified by the two angles required the specify a point on the unit Bloch sphere. It is interesting to note the various transformations in representations, such as the fact that orthogonal states in Hilbert space are antipodal points on the Bloch sphere.

Some general prerequisites the medium in question (here, the quantum system) must satisfy in order to carry encoded information are given: The existence of the computational basis, the exhaustiveness of the computational basis, and the arbitrariness of the computational basis.

Since a qubit can be represented as a superposition of any two arbitrary orthogonal states, it follows that the information associated with a qubit is the same as that contained in an arbitrary superposition, and is independent of information acquisition procedures. Furthermore, since the sphere has infinitely many points, the amount of information contained in the qubit is—infinite!

However, we know that *measurement* results are dependent on measurement acquisition procedures. The following theorem ties up this incongruity, via the fact that we must choose one eigenbasis for measurement.

Holevo theorem: The information that can be acquired from one qubit is at most one bit.

"In other words, when measuring a system, we dump into the environment precisely the non-local features which contribute to the infinite amount of information contained in the qubit." Collapse is reformulated as the gap between information encoding and information acquisition.

Note: 'One bit' is defined as the information entropy of a binary random variable that is 0 or 1 with equal probability (a special case of a qubit). (When the probabilities are unequal, the information that can be acquired becomes less than a bit.)

The follow theorem rules out a possible counter-argument via cloning to the Holevo theorem.

No-cloning theorem: It is impossible to create identical copies of an arbitrary unknown quantum state.

An elementary proof is provided.

Classically, the limitation is the finite resolution of measurement. Quantum mechanically, it is the existence of nonlocal features which are classically un-acquirable. The following sums up our results.

Information accessibility principle: The whole information contained in a system can only be partially accessed.

Information activation principle: Any encoded information is as such potential, and may be activated only by actual (external) conditions.

Information processing is unitary, reversible and requires no energy expenditure. Information acquisition is irreversible and non-unitary, and requires energy expenditure, for it involves (classical) bit-selection.

Having discussed information acquisition briefly, we move on to information processing. Quantum gates are introduced, alongside their unitary matrices. We see how the Mach-Zehnder interferometer can be viewed as a device composed of wires and quantum gates.

Gates introduced: Identity, phase-shift, NOT, Hadamard, CNOT (2-qubit), Toffoli (3-qubit). The last two are the quantum analogues of the classical XOR and Toffoli.

It is shown how we may entangle two systems by successive applications of Hadamard x I and CNOT.

Algorithms, algorithms of polynomial time, algorithms of exponential time (the former being the easy/efficient class), and the class of problems which are checkable but (apparently) insolvable in polynomial time are introduced.

Finally, the Deutsch–Jozsa problem is discussed (for the n=1 case). Classically, it is impossible to determine if the function is constant or balanced in just one run, whereas an explicit method for the same is provided using quantum gates.

Teleportation: An activity based on a quantum channel and a classical channel. Alice has qubits 1 & 2; Bob has 3. 2 & 3 are entangled—an *ebit*. Now, make a measurement which has the effect of entanglement-swapping: 1 & 2 become entangled. The outcome of this measurement is communicated to Bob, who subsequently performs the simple operation required to recover qubit 1 from the final qubit 3 he has. It is noted that in order to teleport one qubit, we require two classical bits and one (shared) ebit (the last one being, it is seen, a *weaker* 'resource' than a qubit). It is noteworthy that neither Alice nor Bob need to know exactly what state qubit 1 is.

Cryptography: It is classically impossible to allow for safe private key distribution. Quantum mechanically, it is. Here is the setup: Alice and Bob perform measurements using two distinct bases. They publicly discuss which basis was used for which measurement and discard the one which don't match. The measurements for which the basis does match will correspond to the same bit for both. If someone intercepts them, the no-cloning theorem, in conjunction with the fact that interaction irreversibly perturbs the state, makes it exponentially likely for the interceptor to be revealed.

Note: The no-cloning theorem is equivalent to the fact that two non-orthogonal states cannot be distinguished between in one measurement.

Having previously defined the Shannon entropy of a system, we now define the *joint entropy* of two systems. The *mutual information* between the two is defined, as a measure of the correlation between the two (and thus, the information shared between the sender and the receiver). The non-negativity of mutual information is provable using real analysis. (Mutual information is zero if the two are perfectly uncorrelated.)

Next, the *conditional entropy* related to two systems is defined (note that only the probability inside the logarithm is perturbed to be conditional). It is rewritten, using the laws of conditional probability, as the difference between the joint entropy and the entropy of the second system. We can also rewrite mutual information in terms of the conditional entropy of the systems (or, more accurately, of the conditional entropy of the second system).

Upon rewriting mutual information in terms of the probability distributions, we see that it is a symmetric quantity (is unchanged if J,K becomes K,J). This indicates that mutual information

represents also that information which is shared in the absence of communication (since the last expression is equivalent to subtracting from the total information of J the communicated information to K, which this symmetry implies is the same as the non-communicated information by K to J).

A Venn diagram neatly sums up the interrelationships between individual entropies, joint entropy, conditional entropies and mutual information. We see that joint entropy is also the sum of the conditional entropies and the mutual information—a *mix* of order and disorder (for mutual information represents orderliness). Another evocative equation is the one saying the total entropy (information) of a system equals the sum of the conditional entropy (the information contained in it given another that system is known) and the mutual information (between the two systems).

Now, all of this was classical—what of the quantum-mechanical counterpart?

"In particular, the quantum mechanical counterpart of the mutual information can be used to quantify the entanglement between two quantum systems."

The "degree" of entanglement between two systems can be equal to the sum of the von Neumann entropies of the two, minus the joint von Neumann entropy. More specifically, this can be done for two chosen observables (one of each system). Here, the joint entropy quantifies the information accessible by a joint measurement of the two observables (i.e., separability); subtracting it out leaves that information which is accessible without having to do a joint measurement (thanks to non-separability, here). Finally, we note that E (O1, O2) cannot exceed E (1,2).

(Note that usually, the von Neumann entropy consists of the eigenvalues of the system's density matrix. When we speak of the same with respect to some observable, we shift from using the spectral decomposition of the density matrix in its own eigenstates to the same in the eigenstates of the observable.)

Classically, the expression for mutual information in terms of joint entropy and conditional entropy were equivalent. This is not true quantum mechanically; the latter is understood as just the classical correlations. Furthermore, the difference between the two has been termed 'quantum discord'.

Using the projection postulate, the state of the system given the state of the apparatus is written; subsequently, the conditional von Neumann entropy (which is: given A, that of S) is written, taking the weighted average of each of the possible outcome-states for the given measurement (characterized by the set of projectors). Finally, it is said that quantum discord is typically always nonnegative.

The classical correlation "is essentially the Holevo bound, i.e., the maximum amount of classical information that can be extracted from a system." Since it depends on the set of projectors we have chosen, one typically tries to make it such that discord is minimized. This corresponds to making the gentlest possible measurement; nonzero discord indicates non-commutativity. Entanglement is a sum of quantum discord and classical correlations.

The complementarity between local irreversibility and global reversibility has an equivalence with the one between classical correlations and quantum discord (respectively) (collapse transfers value from the latter to the former).

"The negativity of quantum conditional entropy is a sufficient criterion for quantum non-separability." (Classically, conditional entropy cannot be negative.)

Expectation value products are replaced by conditional entropies in order to formulate the information-theoretic Bell inequality. Since the entropies here are to be formulated within a local hidden variable framework, we suppose the usual relation between conditional & joint entropy to hold.

The final inequality is derivable in two different ways. The first essentially uses two facts: That the conditional entropy of a system cannot exceed its net entropy; and that the net entropy of a system cannot exceed its joint entropy (with the other). The inequality is then written in terms of entropy.

The second method uses the notion of "information distance". This is the sum of two interchanged conditional entropies, and measures the lack of correlation between the observables. The inequality is then written in terms of information distance.

(The takeaway from the violation of the inequality is that two observables may be less correlated in quantum mechanics than they would in a local hidden variable theory.)

Chapter 12

Information acquisition principle: We can receive information about objects and events only conditionally on the data at our disposal.

A classical analysis is performed on the probability of the value/parameter sought for being *k* if (conditional) we select the event *j* given the data *d*. This is, in fact, the classical analogue of measurement. Quantum mechanically, we have, in the case of full entanglement, a perfect correlation between the parameter and the data.

Supposing full entanglement to have occurred, we want to find the quantum mechanical counterpart for the conditional probability, that is, the probability of event-selection *j* given that the parameter is *k*.

We first have premeasurement, followed by unitary evolution (the environment is ignored). The probability of a given outcome on the apparatus is written using density matrix formalism (and quickly verified). This is then rewritten in a less bulky form using the newly-defined *effect operator.*

It is easy to see that the effect operator is Hermitian, positive semidefinite and satisfies the completeness relation. However, it does not, in general, satisfy orthogonality.

The effect operator is next rewritten in terms of the *amplitude operator*. The latter describes all 3 steps of measurement: Initial state, premeasurement + unitary evolution, final selection. This is summarized by writing out the state of the apparatus post-collapse (using the projection postulate) in terms of the amplitude operator.

Ultimately, all of this is combined to make explicit the quantum-mechanical conditional probability. (The *conditional* aspect appears in the amplitude operator, wherein the ket-state is the system state *k* and the bra-state is the outcome-state *j.*)

Note: The essential distinction between classical conditional probability and quantum conditional probability occurs due to the existence of basis-selection in the latter.

Question: How non-local (or rather, non-separable?) can quantum mechanics be?

Beginning from the CHSH inequality but using only the requirement of locality gives us another inequality. Now, we have seen that quantum-mechanical correlations allow the violation of the similar but stronger CHSH inequality. Is there a number between 2 and 4 which applies the appropriate limit?

Yes, and the number is given by Tsirelson's theorem: 2*sqrt2.

This is proven shortly by cleverly defining new Hermitian operators and using the fact that their expectation values will always be nonnegative.

"The importance of the Tsirelson theorem lies in the fact that it proves that quantum mechanics does *not* fill the entire gap between the two bounds set. The former inequality sets bound 2 for classical separable theories while quantum mechanics satisfies a bound stricter than the bound 4 imposed by locality. There is a wide spectrum of hyper-correlations satisfied by the latter bound, but are nevertheless not allowed by quantum mechanics."

The point—which is to say, the distinctions between the classical case, the quantum case, the hyper-correlation case (and the non-locality case?)—is made clearer by the definition of the numerical parameter D.

Information causality principle: The information gain that Bob can reach about a previously unknown to him data set of Alice, by using all his local resources (which may be correlated with Alice's resources) and *m* classical bits communicated by Alice, is at most *m* bits.

An easy example of this is seen in teleportation, wherein 2 bits are communicated to Bob, but only 1 bit can be gained from the qubit he gets.

The importance of this principle lies in the fact that it is maintained up to the quantum case, and violated in the hyper-correlation case! More clearly, the hyper-correlation case and the violation of the principle signifies correlations between possible apparatus settings (although ones which do not violate relativity), while quantum mechanics only contains correlations between possible outcomes (the former is what deals with a choice of basis).

In a nutshell, non-local encoding of information is forbidden in quantum mechanics.

A final example: In entanglement-correlation, we can only know what outcome the other person gets conditional on their choosing the right basis/ 'input'/ context, but not the latter itself.

We close with three sections bringing out the three ontological entities in quantum mechanics in various different ways. Classically, they amount to the same thing; quantum non-separability is what refines our distinguishing abilities.

1. Preparation, premeasurement, selection
2. State, observable, property
3. [Equivalence class of] Preparations, premeasurements, events
4. Processor, regulator, decider
5. Coupling, selecting, inferring
6. Information processing, information sharing, information selection
7. Unitary evolution, entanglement, non-unitary collapse